

## Exercise 2.1

### Question 1:

If  $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ , find the values of  $x$  and  $y$ .

Answer

It is given that  $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ .

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore,  $\frac{x}{3}+1 = \frac{5}{3}$  and  $y-\frac{2}{3} = \frac{1}{3}$ .

$$\frac{x}{3}+1 = \frac{5}{3}$$

$$\Rightarrow \frac{x}{3} = \frac{5}{3} - 1 \quad y - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{3} = \frac{2}{3} \quad \Rightarrow y = \frac{1}{3} + \frac{2}{3}$$

$$\Rightarrow x = 2 \quad \Rightarrow y = 1$$

$\therefore x = 2$  and  $y = 1$

### Question 2:

If the set  $A$  has 3 elements and the set  $B = \{3, 4, 5\}$ , then find the number of elements in  $(A \times B)$ ?

Answer

It is given that set  $A$  has 3 elements and the elements of set  $B$  are 3, 4, and 5.

$\Rightarrow$  Number of elements in set  $B = 3$

Number of elements in  $(A \times B)$

$=$  (Number of elements in  $A$ )  $\times$  (Number of elements in  $B$ )

$= 3 \times 3 = 9$

Thus, the number of elements in  $(A \times B)$  is 9.

### Question 3:

If  $G = \{7, 8\}$  and  $H = \{5, 4, 2\}$ , find  $G \times H$  and  $H \times G$ .

Answer

$G = \{7, 8\}$  and  $H = \{5, 4, 2\}$

We know that the Cartesian product  $P \times Q$  of two non-empty sets  $P$  and  $Q$  is defined as

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

$\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

### Question 4:

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then  $P \times Q = \{(m, n), (n, m)\}$ .

(ii) If  $A$  and  $B$  are non-empty sets, then  $A \times B$  is a non-empty set of ordered pairs  $(x, y)$  such that  $x \in A$  and  $y \in B$ .

(iii) If  $A = \{1, 2\}$ ,  $B = \{3, 4\}$ , then  $A \times (B \cap \Phi) = \Phi$ .

Answer

(i) False

If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then

$P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$

(ii) True

(iii) True

### Question 5:

If  $A = \{-1, 1\}$ , find  $A \times A \times A$ .

Answer

It is known that for any non-empty set  $A$ ,  $A \times A \times A$  is defined as

$A \times A \times A = \{(a, b, c) : a, b, c \in A\}$

It is given that  $A = \{-1, 1\}$

$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$   
 $(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

### Question 6:

If  $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$ . Find  $A$  and  $B$ .

Answer

It is given that  $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets  $P$  and  $Q$  is defined as  $P \times Q = \{(p, q) : p \in P, q \in Q\}$

$\therefore A$  is the set of all first elements and  $B$  is the set of all second elements.

Thus,  $A = \{a, b\}$  and  $B = \{x, y\}$

### Question 7:

Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$ . Verify that

(i)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii)  $A \times C$  is a subset of  $B \times D$

Answer

(i) To verify:  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have  $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$  L.H.S. =  $A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore$  R.H.S. =  $(A \times B) \cap (A \times C) = \Phi$

$\therefore$  L.H.S. = R.H.S

Hence,  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify:  $A \times C$  is a subset of  $B \times D$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7),$   
 $(3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

We can observe that all the elements of set  $A \times C$  are the elements of set  $B \times D$ .

Therefore,  $A \times C$  is a subset of  $B \times D$ .

**Question 8:**

Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$ . Write  $A \times B$ . How many subsets will  $A \times B$  have? List them.

Answer

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

$$\therefore A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$\Rightarrow n(A \times B) = 4$$

We know that if  $C$  is a set with  $n(C) = m$ , then  $n[P(C)] = 2^m$ .

Therefore, the set  $A \times B$  has  $2^4 = 16$  subsets. These are

$$\Phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\}, \{(1, 3), (1, 4)\}, \{(1, 3), (2, 3)\},$$

$$\{(1, 3), (2, 4)\}, \{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3), (2, 4)\},$$

$$\{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4), (2, 4)\}, \{(1, 3), (2, 3), (2, 4)\},$$

$$\{(1, 4), (2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

**Question 9:**

Let  $A$  and  $B$  be two sets such that  $n(A) = 3$  and  $n(B) = 2$ . If  $(x, 1)$ ,  $(y, 2)$ ,  $(z, 1)$  are in  $A \times B$ , find  $A$  and  $B$ , where  $x$ ,  $y$  and  $z$  are distinct elements.

Answer

It is given that  $n(A) = 3$  and  $n(B) = 2$ ; and  $(x, 1)$ ,  $(y, 2)$ ,  $(z, 1)$  are in  $A \times B$ .

We know that  $A =$  Set of first elements of the ordered pair elements of  $A \times B$

$B =$  Set of second elements of the ordered pair elements of  $A \times B$ .

$\therefore x, y,$  and  $z$  are the elements of  $A$ ; and  $1$  and  $2$  are the elements of  $B$ .

Since  $n(A) = 3$  and  $n(B) = 2$ , it is clear that  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ .

**Question 10:**

The Cartesian product  $A \times A$  has 9 elements among which are found  $(-1, 0)$  and  $(0, 1)$ .

Find the set  $A$  and the remaining elements of  $A \times A$ .

Answer

We know that if  $n(A) = p$  and  $n(B) = q$ , then  $n(A \times B) = pq$ .

$$\therefore n(A \times A) = n(A) \times n(A)$$

It is given that  $n(A \times A) = 9$

$$\therefore n(A) \times n(A) = 9$$

$$\Rightarrow n(A) = 3$$

The ordered pairs  $(-1, 0)$  and  $(0, 1)$  are two of the nine elements of  $A \times A$ .

We know that  $A \times A = \{(a, a) : a \in A\}$ . Therefore,  $-1, 0,$  and  $1$  are elements of  $A$ .

Since  $n(A) = 3$ , it is clear that  $A = \{-1, 0, 1\}$ .

The remaining elements of set  $A \times A$  are  $(-1, -1), (-1, 1), (0, -1), (0, 0),$

$(1, -1), (1, 0),$  and  $(1, 1)$

## Exercise 2.2

### Question 1:

Let  $A = \{1, 2, 3, \dots, 14\}$ . Define a relation  $R$  from  $A$  to  $A$  by  $R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$ . Write down its domain, codomain and range.

Answer

The relation  $R$  from  $A$  to  $A$  is given as

$$R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$$

$$\text{i.e., } R = \{(x, y): 3x = y, \text{ where } x, y \in A\}$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

The domain of  $R$  is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3, 4\}$$

The whole set  $A$  is the codomain of the relation  $R$ .

$$\therefore \text{Codomain of } R = A = \{1, 2, 3, \dots, 14\}$$

The range of  $R$  is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{3, 6, 9, 12\}$$

### Question 2:

Define a relation  $R$  on the set  $\mathbf{N}$  of natural numbers by  $R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4; x, y \in \mathbf{N}\}$ . Depict this relationship using roster form. Write down the domain and the range.

Answer

$$R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbf{N}\}$$

The natural numbers less than 4 are 1, 2, and 3.

$$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$$

The domain of  $R$  is the set of all first elements of the ordered pairs in the relation.

$$\therefore \text{Domain of } R = \{1, 2, 3\}$$

The range of  $R$  is the set of all second elements of the ordered pairs in the relation.

$$\therefore \text{Range of } R = \{6, 7, 8\}$$

### Question 3:

$A = \{1, 2, 3, 5\}$  and  $B = \{4, 6, 9\}$ . Define a relation  $R$  from  $A$  to  $B$  by  $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$ . Write  $R$  in roster form.

Answer

$$A = \{1, 2, 3, 5\} \text{ and } B = \{4, 6, 9\}$$

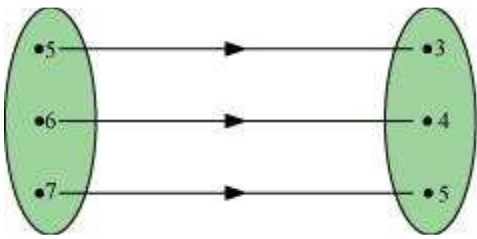
$$R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$$

$$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

### Question 4:

The given figure shows a relationship between the sets  $P$  and  $Q$ . write this relation (i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer

According to the given figure,  $P = \{5, 6, 7\}$ ,  $Q = \{3, 4, 5\}$

(i)  $R = \{(x, y) : y = x - 2; x \in P\}$  or  $R = \{(x, y) : y = x - 2 \text{ for } x = 5, 6, 7\}$

(ii)  $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of  $R = \{5, 6, 7\}$

Range of  $R = \{3, 4, 5\}$

### Question 5:

Let  $A = \{1, 2, 3, 4, 6\}$ . Let  $R$  be the relation on  $A$  defined by

$\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$ .

(i) Write  $R$  in roster form

(ii) Find the domain of  $R$

(iii) Find the range of  $R$ .

Answer

$A = \{1, 2, 3, 4, 6\}$ ,  $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

(i)  $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(ii) Domain of  $R = \{1, 2, 3, 4, 6\}$

(iii) Range of  $R = \{1, 2, 3, 4, 6\}$

### Question 6:

Determine the domain and range of the relation  $R$  defined by  $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$ .

Answer

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$\therefore R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$  Domain of  $R = \{0, 1, 2, 3, 4, 5\}$

Range of  $R = \{5, 6, 7, 8, 9, 10\}$

### Question 7:

Write the relation  $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$  in roster form.

Answer

$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

### Question 8:

Let  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ . Find the number of relations from  $A$  to  $B$ .

Answer

It is given that  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ .

$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$

Since  $n(A \times B) = 6$ , the number of subsets of  $A \times B$  is  $2^6$ .

Therefore, the number of relations from A to B is  $2^6$ .

**Question 9:**

Let R be the relation on  $\mathbf{Z}$  defined by  $R = \{(a, b): a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$ . Find the domain and range of R.

Answer

$R = \{(a, b): a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

$\therefore$  Domain of R =  $\mathbf{Z}$

Range of R =  $\mathbf{Z}$

## Exercise 2.3

### Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i)  $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

(ii)  $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

(iii)  $\{(1, 3), (1, 5), (2, 5)\}$

Answer

(i)  $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain =  $\{2, 5, 8, 11, 14, 17\}$  and range =  $\{1\}$

(ii)  $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain =  $\{2, 4, 6, 8, 10, 12, 14\}$  and range =  $\{1, 2, 3, 4, 5, 6, 7\}$

(iii)  $\{(1, 3), (1, 5), (2, 5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

### Question 2:

Find the domain and range of the following real function:

(i)  $f(x) = -|x|$  (ii)  $f(x) = \sqrt{9-x^2}$

Answer

(i)  $f(x) = -|x|, x \in \mathbf{R}$

We know that  $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

Since  $f(x)$  is defined for  $x \in \mathbf{R}$ , the domain of  $f$  is  $\mathbf{R}$ .

It can be observed that the range of  $f(x) = -|x|$  is all real numbers except positive real numbers.

$\therefore$  The range of  $f$  is  $(-\infty, 0]$ .

(ii)  $f(x) = \sqrt{9-x^2}$

Since  $\sqrt{9-x^2}$  is defined for all real numbers that are greater than or equal to  $-3$  and less than or equal to  $3$ , the domain of  $f(x)$  is  $\{x : -3 \leq x \leq 3\}$  or  $[-3, 3]$ .

For any value of  $x$  such that  $-3 \leq x \leq 3$ , the value of  $f(x)$  will lie between 0 and 3.

$\therefore$  The range of  $f(x)$  is  $\{x : 0 \leq x \leq 3\}$  or  $[0, 3]$ .

### Question 3:

A function  $f$  is defined by  $f(x) = 2x - 5$ . Write down the values of

(i)  $f(0)$ , (ii)  $f(7)$ , (iii)  $f(-3)$

Answer

The given function is  $f(x) = 2x - 5$ .

Therefore,

(i)  $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii)  $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii)  $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

#### Question 4:

The function 't' which maps temperature in degree Celsius into temperature in degree

$$t(C) = \frac{9C}{5} + 32$$

Fahrenheit is defined by

Find (i)  $t(0)$  (ii)  $t(28)$  (iii)  $t(-10)$  (iv) The value of C, when  $t(C) = 212$

Answer

$$t(C) = \frac{9C}{5} + 32$$

The given function is

Therefore,

(i)  $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii)  $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$

(iii)  $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) It is given that  $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$

$$\Rightarrow \frac{9C}{5} = 212 - 32$$

$$\Rightarrow \frac{9C}{5} = 180$$

$$\Rightarrow 9C = 180 \times 5$$

$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Thus, the value of t, when  $t(C) = 212$ , is 100.

#### Question 5:

Find the range of each of the following functions.

(i)  $f(x) = 2 - 3x$ ,  $x \in \mathbf{R}$ ,  $x > 0$ .

(ii)  $f(x) = x^2 + 2$ ,  $x$ , is a real number.

(iii)  $f(x) = x$ ,  $x$  is a real number

Answer

(i)  $f(x) = 2 - 3x$ ,  $x \in \mathbf{R}$ ,  $x > 0$



The values of  $f(x)$  for various values of real numbers  $x > 0$  can be written in the tabular form as

$x$	0.01	0.1	0.9	1	2	2.5	4	5	...
$f(x)$	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

Thus, it can be clearly observed that the range of  $f$  is the set of all real numbers less than 2.

i.e., range of  $f = (-\infty, 2)$

**Alter:**

Let  $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$  Range of  $f = (-\infty, 2)$

(ii)  $f(x) = x^2 + 2$ ,  $x$ , is a real number

The values of  $f(x)$  for various values of real numbers  $x$  can be written in the tabular form as

$x$	0	$\pm 0.3$	$\pm 0.8$	$\pm 1$	$\pm 2$	$\pm 3$	...
$f(x)$	2	2.09	2.64	3	6	11	.....

Thus, it can be clearly observed that the range of  $f$  is the set of all real numbers greater than 2.

i.e., range of  $f = [2, \infty)$

**Alter:**

Let  $x$  be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$  Range of  $f = [2, \infty)$

(iii)  $f(x) = x$ ,  $x$  is a real number

It is clear that the range of  $f$  is the set of all real numbers.

$\therefore$  Range of  $f = \mathbf{R}$

**Question 1:**

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

The relation  $f$  is defined by

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

The relation  $g$  is defined by

Show that  $f$  is a function and  $g$  is not a function.

Answer

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

The relation  $f$  is defined as

It is observed that for

$$0 \leq x < 3, f(x) = x^2$$

$$3 < x \leq 10, f(x) = 3x$$

$$\text{Also, at } x = 3, f(x) = 3^2 = 9 \text{ or } f(x) = 3 \times 3 = 9$$

$$\text{i.e., at } x = 3, f(x) = 9$$

Therefore, for  $0 \leq x \leq 10$ , the images of  $f(x)$  are unique.

Thus, the given relation is a function.

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

The relation  $g$  is defined as

$$\text{It can be observed that for } x = 2, g(x) = 2^2 = 4 \text{ and } g(x) = 3 \times 2 = 6$$

Hence, element 2 of the domain of the relation  $g$  corresponds to two different images

i.e., 4 and 6. Hence, this relation is not a function.

**Question 2:**

$$\frac{f(1.1) - f(1)}{(1.1 - 1)}$$

If  $f(x) = x^2$ , find

Answer

$$f(x) = x^2$$

$$\therefore \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

**Question 3:**

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

Find the domain of the function

Answer

$$\text{The given function is } f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that function  $f$  is defined for all real numbers except at  $x = 6$  and  $x = 2$ . Hence, the domain of  $f$  is  $\mathbf{R} - \{2, 6\}$ .

#### Question 4:

Find the domain and the range of the real function  $f$  defined by  $f(x) = \sqrt{(x-1)}$ .  
 Answer

The given real function is  $f(x) = \sqrt{x-1}$ .

It can be seen that  $\sqrt{x-1}$  is defined for  $(x-1) \geq 0$ .

i.e.,  $f(x) = \sqrt{(x-1)}$  is defined for  $x \geq 1$ .

Therefore, the domain of  $f$  is the set of all real numbers greater than or equal to 1 i.e., the domain of  $f = [1, \infty)$ .

As  $x \geq 1 \Rightarrow (x-1) \geq 0 \Rightarrow \sqrt{x-1} \geq 0$

Therefore, the range of  $f$  is the set of all real numbers greater than or equal to 0 i.e., the range of  $f = [0, \infty)$ .

#### Question 5:

Find the domain and the range of the real function  $f$  defined by  $f(x) = |x-1|$ .

Answer

The given real function is  $f(x) = |x-1|$ .

It is clear that  $|x-1|$  is defined for all real numbers.

∴ Domain of  $f = \mathbf{R}$

Also, for  $x \in \mathbf{R}$ ,  $|x-1|$  assumes all real numbers.

Hence, the range of  $f$  is the set of all non-negative real numbers.

#### Question 6:

$$f = \left\{ \left( x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$

Let  $f$  be a function from  $\mathbf{R}$  into  $\mathbf{R}$ . Determine the range of  $f$ .

Answer

$$f = \left\{ \left( x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$

$$= \left\{ (0, 0), \left( \pm 0.5, \frac{1}{5} \right), \left( \pm 1, \frac{1}{2} \right), \left( \pm 1.5, \frac{9}{13} \right), \left( \pm 2, \frac{4}{5} \right), \left( 3, \frac{9}{10} \right), \left( 4, \frac{16}{17} \right), \dots \right\}$$

The range of  $f$  is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of  $f = [0, 1)$

#### Question 7:

Let  $f, g: \mathbf{R} \rightarrow \mathbf{R}$  be defined, respectively by  $f(x) = x + 1$ ,  $g(x) = 2x - 3$ . Find  $f + g$ ,  $f - g$

$\frac{f}{g}$

and  $\frac{f}{g}$ .

Answer

$f, g: \mathbf{R} \rightarrow \mathbf{R}$  is defined as  $f(x) = x + 1$ ,  $g(x) = 2x - 3$

$$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$$

$$\therefore (f + g)(x) = 3x - 2$$

$$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$$

$$\therefore (f - g)(x) = -x + 4$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, 2x-3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$$

### Question 8:

Let  $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$  be a function from  $\mathbf{Z}$  to  $\mathbf{Z}$  defined by  $f(x) = ax + b$ , for some integers  $a, b$ . Determine  $a, b$ .

Answer

$$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$$

$$f(x) = ax + b$$

$$(1, 1) \in f$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow a \times 1 + b = 1$$

$$\Rightarrow a + b = 1$$

$$(0, -1) \in f$$

$$\Rightarrow f(0) = -1$$

$$\Rightarrow a \times 0 + b = -1$$

$$\Rightarrow b = -1$$

On substituting  $b = -1$  in  $a + b = 1$ , we obtain  $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$ .

Thus, the respective values of  $a$  and  $b$  are 2 and -1.

### Question 9:

Let  $R$  be a relation from  $\mathbf{N}$  to  $\mathbf{N}$  defined by  $R = \{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$ . Are the following true?

(i)  $(a, a) \in R$ , for all  $a \in \mathbf{N}$  (ii)  $(a, b) \in R$ , implies  $(b, a) \in R$

(iii)  $(a, b) \in R$ ,  $(b, c) \in R$  implies  $(a, c) \in R$ .

Justify your answer in each case.

Answer

$$R = \{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$$

(i) It can be seen that  $2 \in \mathbf{N}$ ; however,  $2 \neq 2^2 = 4$ .

Therefore, the statement " $(a, a) \in R$ , for all  $a \in \mathbf{N}$ " is not true.

(ii) It can be seen that  $(9, 3) \in R$  because  $9, 3 \in \mathbf{N}$  and  $9 = 3^2$ .

Now,  $3 \neq 9^2 = 81$ ; therefore,  $(3, 9) \notin R$

Therefore, the statement " $(a, b) \in R$ , implies  $(b, a) \in R$ " is not true.

(iii) It can be seen that  $(9, 3) \in R$ ,  $(16, 4) \in R$  because  $9, 3, 16, 4 \in \mathbf{N}$  and  $9 = 3^2$  and  $16 = 4^2$ .

Now,  $9 \neq 4^2 = 16$ ; therefore,  $(9, 4) \notin \mathbf{N}$

Therefore, the statement " $(a, b) \in R$ ,  $(b, c) \in R$  implies  $(a, c) \in R$ " is not true.

### Question 10:

Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{1, 5, 9, 11, 15, 16\}$  and  $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ . Are the following true?

(i)  $f$  is a relation from  $A$  to  $B$  (ii)  $f$  is a function from  $A$  to  $B$ .

Justify your answer in each case.

Answer

$A = \{1, 2, 3, 4\}$  and  $B = \{1, 5, 9, 11, 15, 16\}$

$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$

It is given that  $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set  $A$  to a non-empty set  $B$  is a subset of the Cartesian product  $A \times B$ .

It is observed that  $f$  is a subset of  $A \times B$ .

Thus,  $f$  is a relation from  $A$  to  $B$ .

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation  $f$  is not a function.

### Question 11:

Let  $f$  be the subset of  $\mathbf{Z} \times \mathbf{Z}$  defined by  $f = \{(ab, a + b) : a, b \in \mathbf{Z}\}$ . Is  $f$  a function from  $\mathbf{Z}$  to  $\mathbf{Z}$ : justify your answer.

Answer

The relation  $f$  is defined as  $f = \{(ab, a + b) : a, b \in \mathbf{Z}\}$

We know that a relation  $f$  from a set  $A$  to a set  $B$  is said to be a function if every element of set  $A$  has unique images in set  $B$ .

Since  $2, 6, -2, -6 \in \mathbf{Z}$ ,  $(2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$

i.e.,  $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation  $f$  is not a function.

### Question 12:

Let  $A = \{9, 10, 11, 12, 13\}$  and let  $f: A \rightarrow \mathbf{N}$  be defined by  $f(n) =$  the highest prime factor of  $n$ . Find the range of  $f$ .

Answer

$A = \{9, 10, 11, 12, 13\}$

$f: A \rightarrow \mathbf{N}$  is defined as

$f(n) =$  The highest prime factor of  $n$

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

$\therefore f(9) =$  The highest prime factor of 9 = 3

$f(10)$  = The highest prime factor of 10 = 5

$f(11)$  = The highest prime factor of 11 = 11

$f(12)$  = The highest prime factor of 12 = 3

$f(13)$  = The highest prime factor of 13 = 13

The range of  $f$  is the set of all  $f(n)$ , where  $n \in A$ .

$\therefore$  Range of  $f = \{3, 5, 11, 13\}$