

1. x = 13

# KENDRIYA VIDYALAYA SANGATHAN, CHENNAI REGION

#### **CLASS XII-COMMON PRE-BOARD EXAMINATION**

### **Answer key (Mathematics)**

## **Section A**

2. $x + y = 6$	1 mark
$3. \ degree = 3$	1 mark
4. $-\frac{\pi}{3}$	1 mark
5. 0	1 mark
6. 0	1 mark
7. 5	1 mark
8. $x + y + z = 13$	1 mark
9. 110	1 mark
10.66	1 mark
Section B	
11. Proving Reflexive	1 mark
Proving Symmetric	1 mark
Proving Transitive	1½ mark
Conclusion	½ mark
12.	
$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right)$	
$\Rightarrow \tan^{-1} \frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{3} X_{5}^{\frac{1}{5}}} + \tan^{-1} \frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} X_{8}^{\frac{1}{8}}}$	1 mark
$\Rightarrow \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{3}{11}$	1 mark
$\Rightarrow \tan^{-1} \frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} X_{\frac{3}{11}}}$	1 mark
$\Rightarrow \tan^{-1} 1$	
$\Rightarrow \frac{\pi}{4}$	1 mark

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OR

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \left(\frac{\pi}{4}\right)$$

$$\Rightarrow \tan^{-1} \left( \frac{x-1}{x-2} \right) + \tan^{-1} \left( \frac{x+1}{x+2} \right) = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\left(\frac{x-1}{x-2}\right) = \tan^{-1} 1 - \tan^{-1}\left(\frac{x+1}{x+2}\right)$$
 1 mark

By applying formula on the R.H.S.

$$\Rightarrow \tan^{-1}\left(\frac{x-1}{x-2}\right) = \tan^{-1}\left(\frac{1}{2x+3}\right)$$
 1 mark

Applying tan both sides and solving

$$x = \pm \frac{1}{\sqrt{2}}$$
 2 mark 
$$13.x^{13}y^7 = (x+y)^{20}$$

$$\Rightarrow \log(x^{13}y^7) = \log(x+y)^{20}$$

$$\Rightarrow 13logx + 7\log y = 20\log(x+y)$$
 1 mark

Differentiating with respect to x

$$\Rightarrow \frac{13y - 7x}{x(x+y)} = \left(\frac{13y - 7x}{y(x+y)}\right) \frac{dy}{dx}$$
 2 mark

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$
 1 mark

OR

$$\Rightarrow y = \tan^{-1} \frac{\sqrt{1 + \cos 2\theta} - \sqrt{1 - \cos 2\theta}}{\sqrt{1 + \cos 2\theta} + \sqrt{1 - \cos 2\theta}}$$

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$$\Rightarrow y = \tan^{-1}(\frac{1-\tan\theta}{1+\tan\theta}) \Rightarrow y = (\frac{\pi}{4} - \theta)$$

$$\Rightarrow y = (\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2)$$
Let  $z = \cos^{-1}x^2$ 

$$\Rightarrow y = (\frac{\pi}{4} - \frac{1}{2}z)$$

$$\Rightarrow \frac{dy}{dz} = -\frac{1}{2}$$
1 mark

$$= \int \frac{e^x (2\sin 2x \cos 2x - 4)}{2\cos^2 2x} dx$$

$$= \int e^x [\tan 2x - 2\sec^2 2x] dx$$

$$= e^x \tan 2x + c$$
1 ½ mark

$$16.y\sqrt{x^2+1} = \ln(\sqrt{x^2+1} - x)$$

Differentiating with respect to x

$$\Rightarrow y \left(\frac{1}{2\sqrt{x^{2}+1}}\right) 2x + \sqrt{x^{2}+1} \frac{dy}{dx} = \frac{1}{\sqrt{x^{2}+1}-x}} \left(\frac{1}{2\sqrt{x^{2}+1}} 2x - 1\right) \quad 1 \text{ mark}$$

$$\Rightarrow \frac{xy}{\sqrt{x^{2}+1}} + \sqrt{x^{2}+1} \frac{dy}{dx} = \frac{x - \sqrt{x^{2}+1}}{(\sqrt{x^{2}+1})(\sqrt{x^{2}+1}-x)}$$

$$\Rightarrow \frac{xy}{\sqrt{x^{2}+1}} + \sqrt{x^{2}+1} \frac{dy}{dx} = -\frac{1}{\sqrt{x^{2}+1}}$$

$$\Rightarrow \sqrt{x^{2}+1} \frac{dy}{dx} = -\frac{(1+xy)}{\sqrt{x^{2}+1}}$$
1 mark
$$\Rightarrow (x^{2}+1) \frac{dy}{dx} + xy + 1 = 0$$
1 mark

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 $17.\int_{-1}^{\frac{3}{2}} |x| \sin \pi x |dx|$ 

$$= \int_{-1}^{1} |x \sin \pi x| dx + \int_{1}^{\frac{3}{2}} |x \sin \pi x| dx$$
 1 mark

$$= \int_{-1}^{1} x \sin \pi x \, dx - \int_{1}^{\frac{3}{2}} x \sin \pi x \, dx$$
 1 mark

On integrating both integrals on right-hand side, we get

$$= \left[ -\frac{x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2} \right]_{-1}^{1} - \left[ -\frac{x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2} \right]_{1}^{\frac{3}{2}}$$
 1 mark

$$=\frac{3}{\pi}+\frac{1}{\pi^2}$$
 1 mark

$$18.\int \left(\frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\right) dx$$

$$= \int \left( \frac{1}{\sqrt{\sin^3 x \left( \sin x \cos \alpha + \cos x \sin \alpha \right)}} \right) dx$$

$$= \int \left(\frac{1}{\sqrt{\sin^4 x \left(\cos\alpha + \cot x \sin\alpha\right)}}\right) dx$$

$$= \int \left( \frac{\cos e^2 x}{\sqrt{(\cos \alpha + \cot x \sin \alpha)}} \right) dx$$
 2 marks

On substitution of  $cos\alpha + cotxsin\alpha = t$ 

$$= \int \left(-\frac{1}{\sin\alpha\sqrt{t}}\right) dt$$

$$= -\frac{2\sqrt{t}}{\sin\alpha} + C$$
 1 mark

On substitution of t  $t = cos\alpha + cotxsin\alpha$ 

$$= -\left(\frac{2}{\sin\alpha}\right)\sqrt{\frac{\sin\left(x+\alpha\right)}{\sin x}} + C$$
 1 mark

$$19.\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \ \vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}, \ \vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\left| \frac{\vec{a} \times (\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|} \right| = \sqrt{2} - (i)$$

$$\vec{b} + \vec{c} = (2 + \lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$$

$$|\vec{b} + \vec{c}| = \sqrt{\lambda^2 + 4\lambda + 44}$$
 ----- (ii)

1 mark

$$\vec{a}X(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2+\lambda & 6 & -2 \end{vmatrix}$$

$$= -8\hat{i} + (4 + \lambda)\hat{j} + (4 - \lambda)\hat{k}$$
----- (iii)

1 mark

By equation (i), (ii) & (iii)

$$\left| \frac{-8\hat{\imath} + (4+\lambda)\hat{\jmath} + (4-\lambda)\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \right| = \sqrt{2}$$

On solving we will get

$$\lambda = 1$$

1 mark

OR

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \left(\vec{a} + \vec{b}\right)^2 = (-\vec{c})^2$$

1 mark

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{c} \cdot \vec{c}$$

½ mark

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2$$

½ mark

By Substitution of values

$$9 + 25 + 2\vec{a} \cdot \vec{b} = 49$$

$$\Rightarrow \vec{a}.\vec{b} = \frac{15}{2}$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = \frac{15}{2}$$

By Substitution of values

$$cos\theta = \frac{1}{2} \Rightarrow \theta = 60^{\circ}$$

1 mark

20.

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$
 and  $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ 

$$\overrightarrow{a1} = -\hat{\imath} - \hat{\imath} - \hat{k}$$

$$\overrightarrow{a2} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$$

$$\overrightarrow{b1} = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$$

$$\overrightarrow{b2} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$$\overrightarrow{a2} - \overrightarrow{a1} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$$

1 mark

$$\overrightarrow{b1} \times \overrightarrow{b2} = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$$

$$\left| \overrightarrow{b1} \times \overrightarrow{b2} \right| = \sqrt{116}$$

1 mark

Shortest distance = 
$$\left| \frac{(\overline{a2} - \overline{a1}).(\overline{b1} \times \overline{b2})}{|\overline{b1} \times \overline{b2}|} \right|$$

1 mark

$$= \left| -\frac{116}{\sqrt{116}} \right|$$

$$=\sqrt{116}$$

1 mark

OR

Equation of plane passing through (2,1,-1) is

$$a(x-2) + b(y-1) + c(z+1) = 0$$
 ---- (i)

½ mark

(i) Passes through (-1,3,4)

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$$\Rightarrow -3a + 2b + 5c = 0$$
 ---- (ii)

(i) is perpendicular to x - 2y + 4z = 10

$$\Rightarrow a - 2b + 4c = 0$$
 ---- (iii)

1 mark

On solving (ii) and (iii)

$$a = 18, b = 17, c = 4$$

1 mark

From (i) equation of plane

$$18x + 17y + 4z - 49 = 0$$

½ mark

21. Let X is the random variable denoting the number of selected scouts, X

takes values 0, 1, 2.

½ mark

$$P(X=0) = \frac{{}^{20}C_2}{{}^{50}C_2} = \frac{38}{245}$$

½ mark

$$P(X = 1) = \frac{({}^{20}C_1X {}^{30}C_1)}{{}^{50}C_2} = \frac{120}{245}$$

½ mark

$$P(X=2) = \frac{{}^{30}C_2}{{}^{50}C_2} = \frac{87}{245}$$

½ mark

Now mean = 
$$\sum (P_i X_i) = \frac{294}{245}$$

1 mark

**Relevant Value** 

22. 
$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

Applying  $R1 \Rightarrow R1 + R2 + R3$ 

$$= \begin{vmatrix} 3(x+y) & 3(x+y) & 3(x+y) \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

1 mark

$$= 3(x+y) \begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

1 mark

Applying  $C1 \Rightarrow C1 - C3$ ,  $C2 \Rightarrow C2 - C3$ 

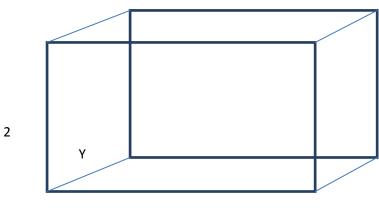
$$= 3(x+y) \begin{vmatrix} 0 & 0 & 1 \\ y & -y & x+y \\ y & 2y & x \end{vmatrix}$$

1 mark

Expanding along R1 we get

$$=9y^2(x+y)$$

1 mark



Χ

23. Diagram

½ mark

Volume of the tank = 
$$8m^3 = 2xy$$
---- (i)

½ mark

Let C be the cost of making the tank

$$C = 70xy + 45 \times 2(2x + 2y)$$

$$C = 70xy + 180(x + y)$$

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From equation (i)

$$C = 70x.\frac{4}{x} + 180(x + \frac{4}{x})$$

$$C = 280 + 180 \left(x + \frac{4}{x}\right)$$
 ---- (ii)

1 mark

$$\frac{dC}{dx} = 180 \left( 1 - \frac{4}{x^2} \right)$$

1 mark

For maxima and minima,  $\frac{dC}{dx} = 0$ 

$$\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0$$

$$\Rightarrow x = \pm 2 \text{ as } x \neq -2, x=2$$

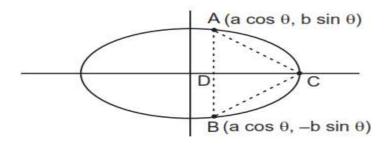
Now 
$$\frac{d^2C}{Dx^2} = 180 X \frac{8}{x^3}$$

$$\left(\frac{d^2C}{Dx^2}\right)_{x=2} = 180 > 0 \Rightarrow C \text{ is minimum at } x = 2$$

By equation (ii)

$$C_{x=2} = Rs. 1000$$

OR



Area of 
$$\triangle ABC = \frac{1}{2} X AB X DC = \frac{1}{2} X 2bsin\theta(a - acos\theta)$$
  
=  $absin\theta(1 - cos\theta)$ -----(i) 1 mark

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$$\frac{dA}{d\theta} = ab(\sin^2\theta + \cos\theta - \cos^2\theta)$$

For maxima and minima  $\frac{dA}{d\theta} = 0$ 

$$ab(\sin^2\theta + \cos\theta - \cos^2\theta) = 0$$

$$\cos\theta - \cos 2\theta = 0$$

$$cos\theta = cos2\theta$$

$$2\theta = 2n\pi \pm \theta$$

$$\theta = n\pi \pm \frac{\theta}{2}$$
----(ii)

As  $\theta \in (0, \pi)$  by equation (ii)

$$\theta = \pi - \frac{\theta}{2}$$

$$\theta = \frac{2\pi}{3}$$

1 mark

$$\frac{d^2A}{d\theta^2} = ab(2\sin 2\theta - \sin \theta)$$

$$\left[\frac{d^2A}{d\theta^2}\right]_{\theta=\frac{2\pi}{3}} < 0 \Rightarrow A \text{ is maximum.}$$

1 mark

By equation (i)

$$A_{\text{max}} = \frac{3\sqrt{3}}{4}ab \text{ sq.unit}$$

1 mark

24.Let x, y, z be the amount of prize to be awarded in the field of agriculture, education & social science respectively. The given situation Can be written in the matrix form as:

$$AX = B$$



Where 
$$A = \begin{bmatrix} 10 & 5 & 15 \\ 15 & 10 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$
,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 70000 \\ 55000 \\ 6000 \end{bmatrix}$ 

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = 75$$

1 mark

$$adj A = \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix}$$

$$A^{-1} = \frac{adj \ A}{|A|} = \frac{1}{75} \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix}$$

2 marks

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{75} \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix} \begin{bmatrix} 70000 \\ 55000 \\ 6000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

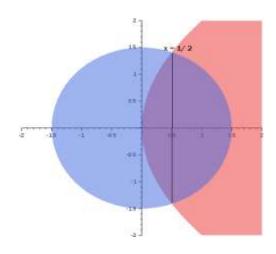
$$\Rightarrow x = 2000, y = 1000, z = 3000$$

1 mark

Value based relevant answer

1 mark

25.





Point of intersection of given curves is  $x = \frac{1}{2}$ 

1 mark

Required Area = 
$$2\left[\int_0^{\frac{1}{2}} \sqrt{4x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9-4x^2}{4}} dx\right]$$

1 ½ mark

$$= 2.2 \left[ x^{\frac{3}{2}} \right]_{0}^{\frac{1}{2}} + 2 \left[ \frac{x}{2} \sqrt{\frac{9 - 4x^{2}}{4}} + \frac{9}{8} \sin^{-1} \left( \frac{2x}{3} \right) \right]_{\frac{1}{2}}^{\frac{3}{2}}$$

1 ½ mark

$$=\frac{9\pi}{8}+\frac{\sqrt{2}}{6}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$$

1 mark

OR

$$I = \int_0^4 (|x - 1| + |x - 2| + |x - 3|) dx ---- (i)$$

$$I = \int_0^4 (|x-1|) dx + \int_0^4 (|x-2|) dx + \int_0^4 (|x-3|) dx$$

1 mark

Let 
$$I_1 = \int_0^4 (|x-1|) dx = \int_0^1 (1-x) dx + \int_1^4 (x-1) dx$$

$$= -\frac{1}{2}[(1-x)^2]_0^1 + \frac{1}{2}[(x-1)^2]_1^4$$

1 ½ mark

$$I_2 = \int_0^4 (|x - 2|) dx = \int_0^2 (2 - x) dx + \int_2^4 (x - 2) dx$$

$$= -\frac{1}{2}[(2-x)^2]_0^2 + \frac{1}{2}[(x-2)^2]_2^4$$

$$=4$$

1 ½ mark

$$I_3 = \int_0^4 (|x-3|) dx = \int_0^3 (3-x) dx + \int_3^4 (x-3) dx$$

$$= -\frac{1}{2}[(3-x)^2]_0^3 + \frac{1}{2}[(x-3)^2]_3^4$$

$$= 5$$

1 ½ mark

By equation (i)

$$I = I_1 + I_2 + I_3 = 5 + 4 + 5 = 14$$

½ mark



26.Let x and y be the number of pieces of type A and B manufactured per week respectively. If z is the profit then,

$$z = 80x + 120y$$

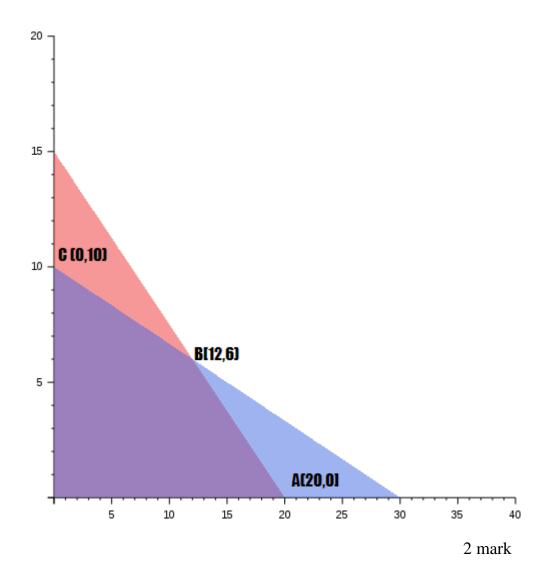
Maximize z subject to the constraint

$$9x + 12y \le 180 \Rightarrow 3x + 4y \le 60$$
----(i)

$$x + 3y \le 30$$
----(ii)

$$x \ge 0, y \ge 0$$
 ----(iii)

2 Mark



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Corner Points	z = 80x + 120y
0(0,0)	0
A(20,0)	1600
B(12,6)	1680 ← maximum
C(0,10)	1200

Relevant answer 1 mark

#### 27.Let the event b defined as

 $E_1$  = The examinee guesses the answer

 $E_2$  = The examinee copies the answer

 $E_3$  = The examinee knows the answer

A = The examinee answers correctly ½ marks

$$P(E_1) = \frac{1}{6}$$
,  $P(E_2) = \frac{1}{9}$ ,  $P(E_3) = 1 - (\frac{1}{6} + \frac{1}{9}) = \frac{13}{18}$ 

½ marks

1 mark

$$P\left(\frac{A}{E_1}\right) = \frac{1}{4}$$
 (Out of 4 choices 1 is correct)

½ marks

$$P\left(\frac{A}{E_2}\right) = \frac{1}{8}$$

$$P\left(\frac{A}{E_3}\right) = 1$$
 (If the answer is known it is always correct)

½ marks

$$P\left(\frac{E_3}{A}\right)$$
 = Required

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3).P\left(\frac{A}{E_3}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

On substitution

$$P\left(\frac{E_3}{4}\right) = \frac{13}{14}$$

1 mark

1 mark

Yes the probability of copying is less than other probability. 1 mark

### 28. The given planes are

$$2x + y - 6z - 3 = 0$$
 ---- (i)

$$5x - 3y + 4z + 9 = 0$$
 ---- (ii)

Equation of the plane passing through the intersection of (i) and (ii)

$$2x + y - 6z - 3 + \lambda(5x - 3y + 4z + 9) = 0 ---- (iii)$$

2 mark

Given that plane (iii) is parallel to  $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{4}$   $\Rightarrow (2+5\lambda).2 + (1-3\lambda).4 + (4\lambda-6).4 = 0$ 

$$\Rightarrow$$
  $(2 + 5\lambda).2 + (1 - 3\lambda).4 + (4\lambda^{2} - 6).4 = 0$ 

1 mark

On solving  $\lambda = \frac{8}{7}$ 

1 ½ mark

On substitution of  $\lambda$  in (iii) equation of plane

$$54x - 17y - 10z + 51 = 0$$

1 ½ mark



$$29.x^{2}dy + y(x + y)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{xy + y^{2}}{x^{2}} - - - - (i)$$

$$\Rightarrow \frac{dy}{dx} = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{vx^{2} + v^{2}x^{2}}{x^{2}}$$

$$\Rightarrow \frac{dv}{v^{2} + 2v} = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \int \left(\frac{1}{v} - \frac{1}{v + 2}\right) dv = \int \left(-\frac{dx}{x}\right)$$

$$\Rightarrow \log|v| - \log|v + 2| = -2\log|x| + C$$

$$\Rightarrow \log\left|\frac{vx^{2}}{v + 2}\right| = \log k$$

$$\frac{vx^{2}}{v + 2} = k$$

$$\Rightarrow \text{Putting } v = \frac{y}{x} \text{ we get}$$

$$x^{2}y = k(y + 2x)$$

$$\Rightarrow \text{Therefore Particular Solution is}$$

$$3x^{2}y = y + 2x$$

$$1 \text{ mark}$$