Maths Class 10 Notes for Quadratic Equations

QUADRATIC EQUATIONS

The polynomial of degree two is called quadratic polynomial and equation corresponding to a quadratic polynomial \( P(x) \) is called a quadratic equation in variable \( x \).

Thus, \( P(x) = ax^2 + bx + c = 0, a \neq 0, a, b, c \in \mathbb{R} \) is known as the standard form of quadratic equation.

There are two types of quadratic equation.
(i) **Complete quadratic equation**: The equation \( ax^2 + bx + c = 0 \) where \( a \neq 0, b \neq 0, c \neq 0 \)
(ii) **Pure quadratic equation**: An equation in the form of \( ax^2 = 0, a \neq 0, b = 0, c = 0 \)

ZERO OF A QUADRATIC POLYNOMIAL

The value of \( x \) for which the polynomial becomes zero is called zero of a polynomial

For instance,

1 is zero of the polynomial \( x^2 — 2x + 1 \) because it become zero at \( x = 1 \).

SOLUTION OF A QUADRATIC EQUATION BY FACTORISATION

A real number \( x \) is called a root of the quadratic equation \( ax^2 + bx + c = 0, a \neq 0 \) if \( a\alpha^2 + b\alpha + c = 0 \). In this case, we say \( x = \alpha \) is a solution of the quadratic equation.

NOTE:

1. The zeroes of the quadratic polynomial \( ax^2 + bx + c \) and the roots of the quadratic equation \( ax^2 + bx + c = 0 \) are the same.
2. Roots of quadratic equation \( ax^2 + bx + c = 0 \) can be found by factorizing it into two linear factors and equating each factor to zero.

SOLUTION OF A QUADRATIC EQUATION BY COMPLETING THE SQUARE

By adding and subtracting a suitable constant, we club the \( x^2 \) and \( x \) terms in the quadratic equation so that they become complete square, and solve for \( x \).

In fact, we can convert any quadratic equation to the form \( (x + a)^2 — b^2 = 0 \) and then we can easily find its roots.

**DISCRIMINANT**

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The expression $b^2 - 4ac$ is called the discriminant of the quadratic equation.

**SOLUTION OF A QUADRATIC EQUATION BY DISCRIMINANT METHOD**

Let quadratic equation is $ax^2 + bx + c = 0$

**Step 1.** Find $D = b^2 - 4ac$.

**Step 2.**

(i) If $D > 0$, roots are given by

$$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$$

(ii) If $D = 0$ equation has equal roots and root is given by $x = -\frac{b}{2a}$.

(iii) If $D < 0$, equation has no real roots.

**ROOTS OF THE QUADRATIC EQUATION**

Let the quadratic equation be $ax^2 + bx + c = 0$ ($a \neq 0$).

Thus, if $b^2 - 4ac \geq 0$, then the roots of the quadratic

$$-b \pm \sqrt{b^2 - 4ac}$$

/ $2a$ equation are given by

**QUADRATIC FORMULA**

$$-b \pm \sqrt{b^2 - 4ac} / 2a$$

is known as the quadratic formula

which is useful for finding the roots of a quadratic equation.

**NATURE OF ROOTS**

(i) If $b^2 - 4ac > 0$, then the roots are **real and distinct**.

(ii) If $b^2 - 4ac = 0$, the roots are **real and equal or coincident**.

(iii) If $b^2 - 4ac < 0$, the roots are not **real (imaginary roots)**

**FORMATION OF QUADRATIC EQUATION WHEN TWO ROOTS ARE GIVEN**

If $\alpha$ and $\beta$ are two roots of equation then the required quadratic equation can be formed as $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

**NOTE :**

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Let \( \alpha \) and \( \beta \) be two roots of the quadratic equation \( ax^2 + bx + c = 0 \) then

**Sum of Roots:** \(- the coefficient of \( x \) / the coefficient \( t \) of \( x^2 \) \( \Rightarrow \alpha + \beta = -\frac{b}{a} \)

**Product of Roots:**
\( \alpha\beta = \text{constant term} / \text{the coefficient } t \text{ of } x^2 \Rightarrow \alpha\beta = \frac{c}{a} \)

**METHOD OF SOLVING WORD PROBLEMS**

**Step 1:** Translating the word problem into Mathematics form (symbolic form) according to the given condition

**Step 2:** Form the word problem into Quadratic equations and solve them.